Application of mathematical equations to describe lactation curves in buffaloes

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Abstract

The present study was carried out to investigate the efficacy of fitting three different lactation curve models using monthly milk yield records of Marathwadi buffaloes maintained at Cattle Breeding Farm, College of Veterinary and Animal Sciences, Udgir Dist- Latur Maharashtra spread over a period of nine years. The monthly milk yield was recorded at 6^{th}, 36^{th}, 66^{th} and so on till standard lactation with thirty days interval. The three different lactation curve models *viz*.exponential decline function, gamma-type function and parabolic exponential were used on monthly milk yields in Marathwadi buffaloes. It was inferred that the highest coefficient of determination for fitting of lactation curves models was with gamma-type function (98.82%), followed by parabolic exponential model (98.60%) and least by exponential decline function (94.79%). Further, the root mean square error was found least for gamma-type function (1.20), followed by parabolic exponential function (1.31) and highest by exponential decline function (2.53). It is therefore inferred that mixed log function fitted best in Marathwadi buffalo for prediction and modeling of lactation curves based upon monthly milk yields.

Key Words: Gamma type function; Lactation curve; Mathematical models; Marathwadi buffalo

Introduction

Buffaloe (*Bubalus bubalis*) is an important source of livelihood for rural people in India and other developing countries. and is now favoured by both rich and landless farmers. Buffaloe is source of milk as well as meat and therefore is favoured by both rich farmers (Chandra Prasad et al 2019) and marginal farmers in tribal belt (Gaur & Purohit 2020). Marathwada region of Maharashtra state is considered as the most drought prone region in India. The cattle and buffalo breeds in this region are well adapted to the harsh climatic conditions. Marathwadi buffalo is only buffalo breed found in this region with larger built and long flat horns. These buffaloes have breeding tract in Marathwada region of Western India especially in Parbhani, Nanded, Beed, Hingoli, Jalna and Latur districts. The average total lactation milk yield across different lactations is 1118 kg and average milk fat is 8.8% ranging from 6.25-10.50% (Kataria, 2012). Milk production evolves during lactation following a cycle that is usually characterized by two different phases: an ascending phase from parturition to peak production (the maximum production) and a downward phase, from this peak to the dry period. The ascending phase consist of almost three to four months and milk production reaches at peack level during this phase. (Masselin *et al.*, 1987). The descending phase of lactation is the longest during which the milk secretion gradually decreases until dry up. This second phase is explained by the involution of secretory cells but especially by the fall of their numbers. Modeling of lactation curves in dairy animals is important for prediction milk yield, nutritional interventions, breeding management etc. (Macciotta *et al.*, 2005). The application of mathematical models on the lactation records can provide important predictive information and predicting the evolution of milk production at the individual or herd level is a powerful tool for overall managing herd performance. Therefore, there is need to find out the best mathematical model for prediction of lactation milk yield in Marathwadi buffaloes. The present investigation was carried out to the objectives to explain lactation curve with different mathematical models and to come out with the good fit for prediction of the lactation milk yield of Marathwadi buffaloes.

Materials & Methods

The monthly milk yields and 305 days milk yield data have been collected from the history-cum- pedigree sheets and daily milk record registers of Marathwadi buffalo maintained at cattle breeding farm, College of Veterinary and Animal Sciences, Udgir Dist- latur Maharashtra spread over a period of 9 years (2008-2016). The monthly milk yield was recorded at 6^{th}, 36^{th}, 66^{th} and so on till standard lactation (lactation length 305 days) with thirty days interval. The records of the animals with normal lactation were considered for this study. Culling, disposal in middle of lactation, abortion, stillbirth and other pathological conditions which affected the lactation yield were considered as abnormalities and hence such records were excluded from analysis. The geographical location of farm lies on 17º35' N latitude and 72º40' E longitude in deccan platue zone. The climate of the farm is semi-arid in nature. The minimum temperature falls to 12-18ºC in winter months, whereas the maximum temperature goes up to 43-45 ºC in summer. The annual rainfall is about 500 to 1100 mm out of which most of the rainfall is received during the months of July and August. Good quality green fodder with *ad lib* along with eight hours free grazing is practices. Concentrate was provided to cows as per their milk production only during milking hours. Four different linear lactation curve models were studied in the present work are as follows:

Exponential decline function: (Brody et al., 1923)

$$Y_t = ae^{-ct}$$

Where,

Y_t = Average daily yield in the t^{th} week of lactation; a = initial milk yield after calving

c = descending slop parameter; t = length of time since calving

The gamma-type function: (Wood, 1967)

$$Y_t = at^b e^{-ct}$$

Where,

Y_t = Average daily yield in the t^{th} week ; a = initial milk yield after calving

b = ascending slope parameter up to the peak yield; c = descending slop parameter

The constants can be derived by solving the above equation after transformation on the log scale

$$In(Y_t) = In(a) + bIn(t) - ct$$

The milk yield up to week t is given by

$$Y_t = a \int_{0to1} t^b \exp(-ct)dt$$

Thus we can get total 305 days milk yield (43 weeks) as the integral of the average weekly milk yields.

Parabolic exponential model: (Sikka, 1950)

$$Y_t = a \exp\left(bt - ct^2\right)$$

Where,

Y_t = Average daily yield in the t^{th} week of lactation; a = initial milk yield after calving

b = ascending slope parameter up to the peak yield; c = descending slop parameter.

These models were fitted on monthly milk yields records for prediction of 305-day milk yields in Marathwadi buffaloes. The most suitable model was identified on the basis of the highest R^2-value and root mean square error (RMSE) which was further used in prediction.

Results and Discussions

Marathwadi buffaloes are low milk yielder. It was observed that the milk production of Marathwadi buffalo is slightly lower with peak monthly milk yield of 129.66 liters in second month of lactation. Geetha *et al*. (2006), Chakraborty *et al*. (2010) and Patil *et al*. (2012) reported the peak of monthly test days milk yield records as 7.15±0.15 kg, 8.11±0.25 kg and 8.02±0.09 kg, respectively in Murrah buffaloes. The persistence phase of the lactation is well maintained with the peak achieved early as compared to the other buffalo breeds which normally achieve the peak yield in 2nd to 4th month of the lactation (Fig. 1). Although, the breed is low yielder but the consistency of milk production is better.

The estimated lactation curves parameters i.e. a, b and c of all three lactation curve models and mathematical models and obtain the predicting equation has been presented in Table 1and 2 respectively.

The gamma type function was explained the best in all stages of the lactations i.e. ascending, peak and descending phases. It explained the highest coefficient of determination (R^2) i.e. 98.82 % with lowest root mean square error (RMSE) i.e. 1.20 (litrs). Throughout lactation both curves i.e. predicted and actual monthly milk yield shows almost overlapped on each other (Figure 2). Contrary to the present findings, the lowest R2 value (96.42%) and highest RMSE value (0.07kg) was obtained by gamma type function was observed by Singh *et al* 2015. However, R^2 value (96%) was reported by Kumar and Bhatt (1979) and Aziz *et al*. (2006) in Indian buffaloes and Egyptian buffaloes, respectively. Sahoo *et al.* (2014) also reported lowest R2 value (93.13%) in Murrah buffaloes using this function.

The exponential decline function was least fitted to on monthly milk yield records in Marathwadi buffaloes with coefficient of determination (R^2) 94.79 % and RMSE 2.53 litrs. However, during mid lactation i.e. 5th and 6th months, this model represent almost actual milk yield (Figure 3). Higher coefficient of determination (98.64 %) with lower RMSE value (0.04kg) was observed by Singh *et al*., 2015 in Murrah buffaloes on monthly test days records.

Parabolic exponential function was second best function fitted on the monthly milk yield records in Marathwadi buffaloes with coefficient of determination as 98.60 % and RMSE of 1.31 litrs (Figure 4). The curve was fitted almost same as that of gamma type function (Figure 5). Savaliya *et al* (2020) observed 95.00 % coefficient of determination in Gir cattle while Dongre *et al (*2013) observed almost similar results in Sahiwal cattle.

Table 1: Estimated lactation curve parameters of different functions

Sr. No	Lactation Model	Parameters		
		a	b	c
1	Gamma type function	-7.719	44.871	-0.812
2	Exponential decline function	62.394	-0.0001	----
3	Parabolic exponential model	114.390	-0.503	-0.00243

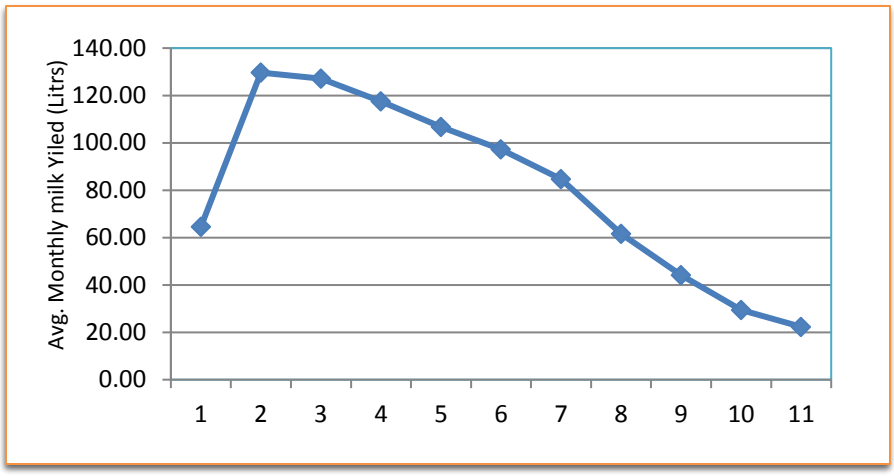

Fig. 1: Average milk yield (litrs) in Marathwadi buffaloes

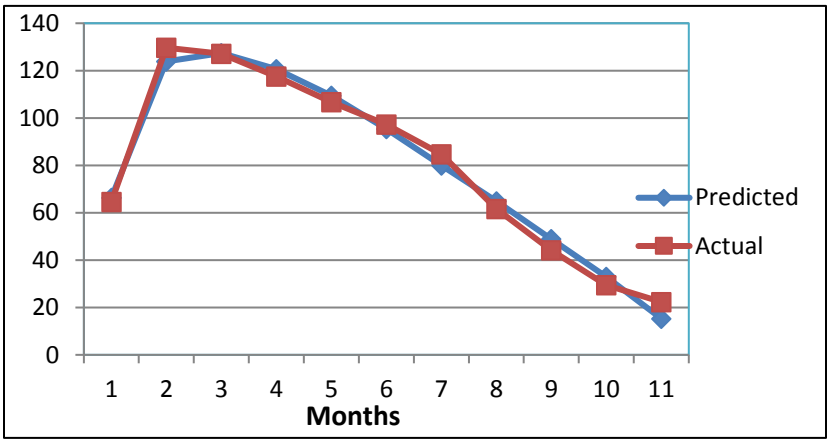

Fig 2: Observed and predicted Monthly milk yield for Gamma type function

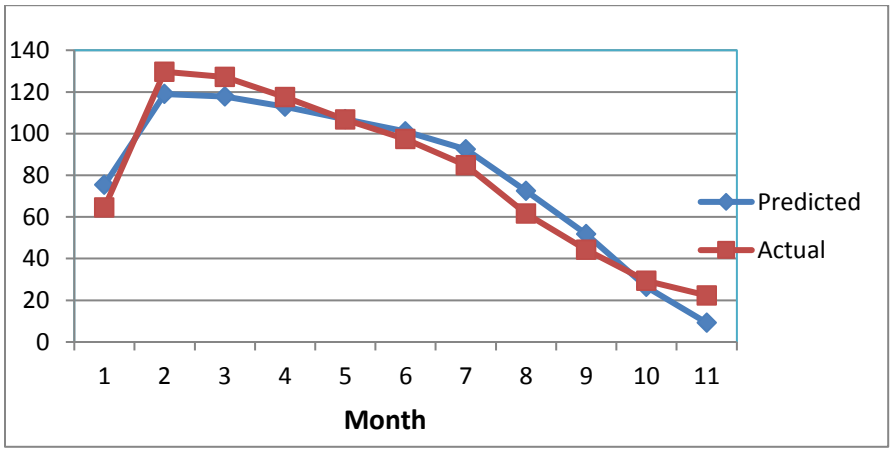

Fig 3: Observed and predicted monthly milk yield for Exponential decline function

Table 2: Different lactation curve models with parameters for prediction of monthly milk yield in Marathwadi buffaloes

S. N.	Models	Parameters of function	R^2 Value	RMSE (Litrs)
1	Gamma type function	$Y_t = (-7.719)t^{44.871}e^{0.812t}$	98.82	1.20
2	Exponential decline function	$Y_t = 62.394e^{0.0001t}$	94.79	2.53
3	Parabolic exponential model	$Y_t = 114.390\exp\left(-0.503t + 0.00243t^2\right)$	98.60	1.31

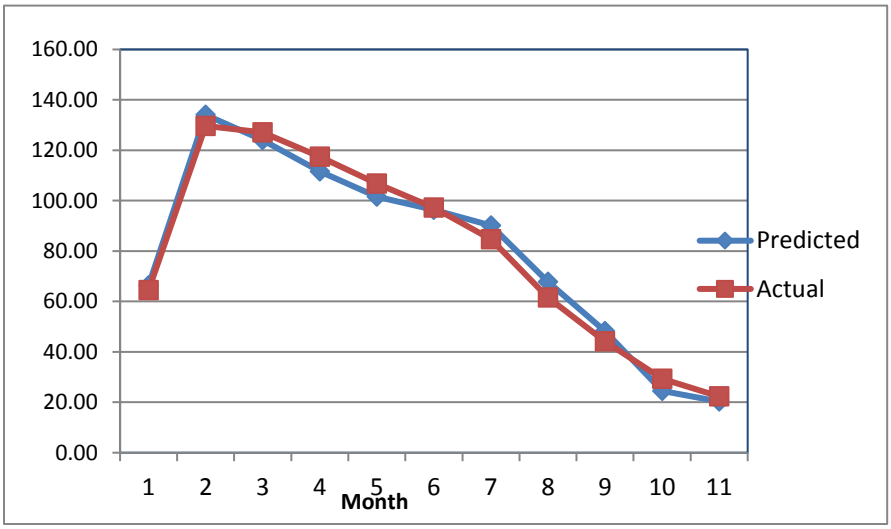

Fig 4: Observed and predicted Monthly milk yield for parabolic exponential function

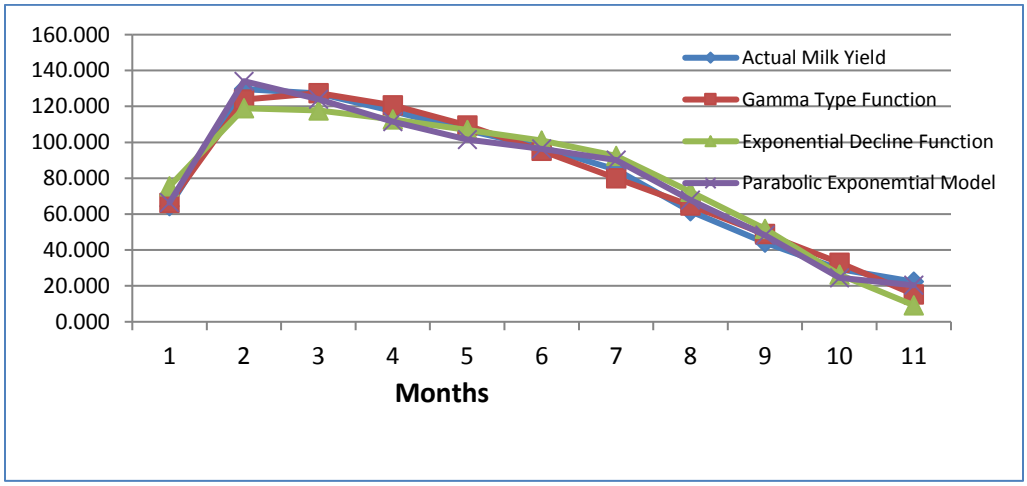

Fig 5: Observed and predicted monthly milk yield for different lactation curve models in Marathwadi Buffaloes

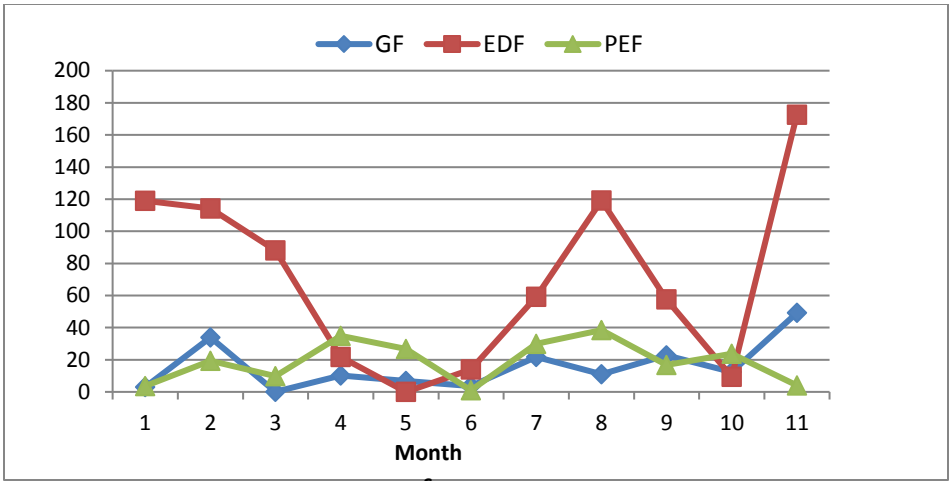

Fig 6: Residuals of the different lactation curve models fitted on monthly milk yield records in Marathwadi Buffaloes

Conclusion

It was observed that the highest coefficient of determination for fitting of lactation curves models was with gamma type function was highest i.e. 98.82 % followed by parabolic exponential models i.e. 98.60 and least was observed in exponential decline function with 94.79%. Gamma type function could explain all stages of lactation i.e. ascending, peak yield, persisteny. Further, the root mean square error was found least for gamma type function (1.20 litrs), followed by parabolic exponential function (1.21 litrs) and highest in exponential decline function (2.53 litrs). Therefore, it is concluded that gamma type function fitted best in Marathwadi buffalo for prediction and modeling of lactation curves based upon monthly milk yields records.

Ethical statement: No Animals were involved in the experiments. Only data on various production trails were collected.

Conflict of interest declaration: All authors hereby declare that there is no conflict of interest regarding the data and manuscript

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